	s	"
Nov. 23	−0·22	−0·2
25	0·25	+0·4
28	0·20	+2·0
30	0·11	+1·0
Dec. 5	0·16	−0·1
10	0·21	+2·2
12	0·04	+2·8
14	0·33	+1·3

The mean of these errors is $-0''\cdot19$ in Right Ascension, and $+1''\cdot2$ in Declination, and therefore less than some of the residual errors of the normal places which have been used for determining the elliptic elements. As the planet was very near the Earth at this last opposition, I trust that my tables will give its place with sufficient accuracy for a long time to come.

I also obtained a few observations of *Vesta* :—

		R.A.	Decl.
	h m s	h m s	° ′ ″
1866 Aug. 25	12 40 8·8	22 56 22·95	−17 34 52·4
Sept. 2	12 1 27·4	22 49 7·61	−18 35 11·0

which give the following errors of the Ephemeris in the Berlin *Jahrbuch* :

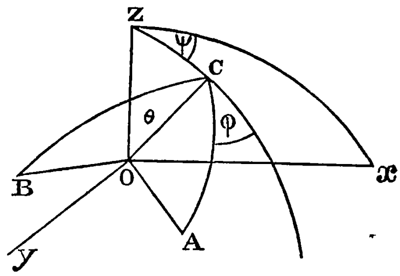
Aug. 25	+3·61	+30·3
Sept. 2	3·92	32·5

These errors, being unusually large, led me to suspect that the Ephemeris which had been computed from my manuscript tables by Dr. Powalky of Berlin was wrong. I therefore computed the place for August 25 myself, and found indeed an error in the Ephemeris of the *Jahrbuch* amounting to $-0''\cdot71$ and $-10''\cdot0$. This correction will decrease the errors given above, but the remaining error is still larger than I expected. It may be, however, that the effect of the terms of the second order which have been neglected in constructing the tables begins to show itself.

On the Possibility of a Change in the Position of the Earth's Axis due to Frictional Action connected with the Phenomena of the Tides. By E. J. Stone, Esq.

I shall first consider that the frictional action is equivalent to a couple whose intensity is proportional to the existing

angular velocity, and whose axis coincides with the instantaneous axis.



Let O be the centre of gravity of the spheroidal Earth; Ox, Oy, Oz, rectangular axes fixed in space; OC, OA, OB, principal axes; ω_3, ω_1, ω_2, the angular velocities about these axes; θ, ψ, and φ, the angles which define their position in space at the time t; C, A, A, the moments of inertia of the spheroid about the same principal axes; $-\mu\,\Omega$, the moment of the couple of resistance about the instantaneous axis.

The equations of motion are

$$\mathrm{A}\,\frac{d\omega_1}{dt} + (\mathrm{C}-\mathrm{A})\,\omega_2\,\omega_3 = -\mu\,\omega_1 \qquad (1)$$

$$\mathrm{A}\,\frac{d\omega_2}{dt} - (\mathrm{C}-\mathrm{A})\,\omega_1\,\omega_3 = -\mu\,\omega_2 \qquad (2)$$

$$\mathrm{C}\,\frac{d\omega_3}{dt} = -\mu\,\omega_3 \qquad (3)$$

I shall write n for $\frac{\mathrm{C}-\mathrm{A}}{\mathrm{A}}$; p for $\frac{\mu}{\mathrm{A}}$; q for $\frac{\mu}{\mathrm{C}}$.

From (3),

$$\omega_3 = \gamma\,\epsilon^{-qt}.$$

From (1) and (2),

$$\frac{\omega_1\,d\omega_1 + \omega_2\,d\omega_2}{\omega_1^2 + \omega_2^2} = -p\,dt,$$

or

$$\omega_1^2 + \omega_2^2 = \alpha^2\,\epsilon^{-2pt}.$$

Substituting in (1) we obtain

$$\frac{d\omega_1}{dt} + n\,\gamma\,\epsilon^{-qt}\sqrt{\alpha^2\,\epsilon^{-2pt} - \omega_1^2} = -p\,\omega_1$$

$$\frac{d\,(\omega_1\,\epsilon^{pt})}{dt} + n\,\gamma\,\epsilon^{-qt}\sqrt{\alpha^2 - (\omega_1\,\epsilon^{pt})^2} = 0$$

$$\frac{d\,(\omega_1\,\epsilon^{pt})}{\sqrt{\alpha^2 - (\omega_1\,\epsilon^{pt})^2}} + n\,\gamma\,\epsilon^{-qt}\,.\,dt = 0$$

$$\cos^{-1}\frac{\omega_1}{\alpha}\varepsilon^{pt} = \mathrm{P} - n\frac{\gamma}{q}\varepsilon^{-q}$$

$$\omega_1 = \alpha\,\varepsilon^{-pt}\cos\left(\mathrm{P} - n\frac{\gamma}{q}\varepsilon^{-qt}\right) \tag{4}$$

Similarly

$$\omega_2 = \alpha\,\varepsilon^{-pt}\sin\left(\mathrm{P} - n\frac{\gamma}{q}\varepsilon^{-qt}\right) \tag{5}$$

and

$$\omega_3 = \gamma\,\varepsilon^{-qt} \tag{6}$$

$$\Omega^2 = \omega_1^2 + \omega_2^2 + \omega_3^2 = \gamma^2\varepsilon^{-2qt} + \alpha^2\varepsilon^{-2pt}.$$

If λ is the angle between the polar axis of figure and the axis of rotation at time t, we have

$$\cos\lambda = \frac{\omega_3}{\Omega} = \frac{\gamma\,\varepsilon^{-qt}}{\sqrt{\gamma^2\varepsilon^{-2qt} + \alpha^2\varepsilon^{-2pt}}}$$

that is

$$\cos\lambda = \frac{1}{\sqrt{1 + \frac{\alpha^2}{\gamma^2}\varepsilon^{-2(p-q)t}}}$$

or
$$\tan\lambda = \frac{\alpha}{\gamma}\varepsilon^{-(p-q)t} \tag{7}$$

Now $p-q$ is positive, and therefore as t increases positively $\tan\lambda$ becomes more and more nearly equal to zero. Consequently, if our spheroid was at any time rotating about an axis not coincident with a principal axis, it would ever tend to a rotation about the least axis of figure, and after the lapse of ages would be found rotating about an axis but slightly inclined to the least axis of figure. It would appear, therefore, that in this case a near coincidence of the axis of rotation and axis of figure would not in itself be a proof that such near coincidence had always held.

The equations for the determination of the motion of the axes are

$$\frac{d\psi}{dt}\sin\theta = -\omega_1\cos\varphi + \omega_2\sin\varphi \tag{8}$$

$$\frac{d\theta}{dt} = \omega_1\sin\varphi + \omega_2\cos\varphi \tag{9}$$

$$\frac{d\varphi}{dt} + \frac{d\psi}{dt}\cos\theta = \omega_3 \tag{10}$$

Hence

$$\left(\frac{d\theta}{dt}\right)^2 + \left(\frac{d\psi}{dt}\right)^2 \sin^2\theta = \omega_1^2 + \omega_2^2 = \alpha^2 \varepsilon^{-2pt} \qquad (11)$$

Differentiating (9) we obtain the following equation for the determination of θ:—

$$\left(\frac{d^2\theta}{dt^2} + p\frac{d\theta}{dt}\right)\varepsilon^{2pt} + \frac{C}{A}\gamma^2\varepsilon^{(p-q)t}\sqrt{\alpha^2 - \left(\frac{d\theta}{dt}\varepsilon^{pt}\right)^2} - \cot\theta\left(\alpha^2 - \left(\frac{d\theta}{dt}\varepsilon^{pt}\right)^2\right) = 0 \qquad (12)$$

The value of θ is complicated by the diurnal rotation, and I am unable to obtain the complete integral of this equation. If v is the cosine of the angle between Oz and the instantaneous axis at time t, then

$$v\sqrt{\alpha^2\varepsilon^{-2pt} + \gamma^2\varepsilon^{-2qt}} = \omega_3\cos\theta - \omega_1\sin\theta\,.\cos\phi + \omega_2\sin\theta\,.\sin\phi$$

or

$$v\sqrt{\alpha^2\varepsilon^{-2pt} + \gamma^2\varepsilon^{-2qt}} = \gamma\varepsilon^{-qt}\cos\theta + \sin^2\theta\frac{d\psi}{dt}$$

$$= \gamma\varepsilon^{-qt}\cos\theta + \sin\theta\sqrt{\alpha^2\varepsilon^{-2pt} - \left(\frac{d\theta}{dt}\right)^2}$$

Therefore

$$v\sqrt{\alpha^2 + \gamma^2\varepsilon^{2xt}} = \gamma\varepsilon^{xt}\cos\theta + \sin\theta\sqrt{\alpha^2 - \left(\frac{d\theta}{dt}\varepsilon^{pt}\right)^2} \qquad (13)$$

where

$$x = p - q.$$

Differentiating (13) and availing ourselves of (12), we find

$$\frac{d}{dt}(v)\,.\sqrt{\alpha^2 + \gamma^2\varepsilon^{2xt}} + \frac{v\,.\,\gamma\varepsilon^{2xt}x}{\sqrt{\alpha^2 + \gamma^2\varepsilon^{2xt}}} = \gamma x\varepsilon^{xt}\left\{\cos\theta + \frac{\sin\theta}{q}\frac{d\theta}{dt}\right\} \qquad (14)$$

Now

$$\cos\theta = v\cos\lambda + \sqrt{1 - v^2}\,.\sin\lambda\,.\cos \mathrm{ZIP};$$

but $\cos\lambda$ and $\sin\lambda$ contain no periodic terms; v changes with extreme slowness; ZIP changes from 0 to 2π, at the present time in 24^{h}. Hence we here neglect the last term and take

$$\cos\theta = v\cos\lambda = \frac{v}{\sqrt{1 + \frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}}}$$

whence

$$-\sin\theta\frac{d\theta}{dt}=\frac{dv}{dt}\cdot\frac{1}{\sqrt{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}}}+\frac{v\cdot\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}x}{\left(1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}\right)^{\frac{3}{2}}}$$

Therefore (14) becomes

$$\frac{dv}{dt}\sqrt{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}}+\frac{vx}{\sqrt{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}}}=x\left\{\begin{array}{l}\dfrac{v}{\sqrt{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}}}-\dfrac{\frac{v}{q}\,\frac{\alpha^2}{\gamma^2}\,x\,\varepsilon^{-2xt}}{\left(1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}\right)^{\frac{3}{2}}}\\[2ex] -\dfrac{1}{q}\dfrac{dv}{dt}\cdot\dfrac{1}{\sqrt{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}}}\end{array}\right.$$

$$\therefore\ \frac{dv}{dt}\left\{\left(1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}\right)+\frac{x}{q}\right\}=\frac{-\frac{x}{q}\,v\cdot\frac{\alpha^3}{\gamma^2}\varepsilon^{-2xt}\,x}{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}},$$

$$\frac{1}{v}\,\frac{dv}{dt}=\frac{1}{2}\left\{\frac{d\tau}{\left(\tau+\frac{x}{2q}\right)-\frac{x}{2q}}-\frac{d\tau}{\left(\tau+\frac{x}{2q}\right)+\frac{x}{2q}}\right\}$$

where

$$\tau=1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}$$

$$\therefore\ \text{Log}\,\frac{v}{A}=\frac{1}{2}\,\text{Log}\,\frac{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}}{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}+\frac{x}{q}}$$

$$v=\frac{A\sqrt{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}}}{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}+\frac{x}{q}}$$

Determining A so that $v=\dfrac{\gamma}{\sqrt{\alpha^2+\gamma^2}}$ when $t=0$, we have

$$v=\frac{\frac{\gamma^2}{\alpha^2+\gamma^2}\sqrt{1+\frac{\alpha^2}{\gamma^2}+\frac{x}{q}}\sqrt{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}}}{1+\frac{\alpha^2}{\gamma^2}\varepsilon^{-2xt}+\frac{x}{q}}\qquad(16)$$

Let

$v=\cos\mu$ and $\cos\lambda_0=\dfrac{\gamma}{\sqrt{\alpha^2+\gamma^2}}$ $\tan\lambda_0=\dfrac{\alpha}{\gamma}$

Then we have

$$\tan \lambda = \tan \lambda_0 \cdot \varepsilon^{-xt} \tag{17}$$

and

$$\cos \mu = \cos \lambda_0 \frac{\sqrt{1 + \frac{C-A}{A} \cdot \cos^2 \lambda_0}}{1 + \frac{C-A}{A} \cdot \cos^2 \lambda} \tag{18}$$

It appears, therefore, that the instantaneous axis is really subject to secular displacement both in the spheroid and in space. If, however, we reduce these formulæ to numbers, throwing the whole of the outstanding 6″ of the Moon's secular acceleration upon the diminution of the Earth's rotation, it will be found that the secular displacements are on such a scale that it would require thousands of millions of years to produce effects of any magnitude.

If we assume that $\lambda_0 = 6^\circ$, an extreme supposition, and take one million of years as our unit of time, we have

$$\text{Log } q = 5{\cdot}8409759$$

and

$$\frac{C-A}{A} = 0{\cdot}00314578 \text{ nearly.}$$

Then for $t = -1000000 \times$ one million years, we have $\delta \lambda = 3^\circ 51'$.

The change in the value of $\delta \mu$ is quite insignificant.

The assumption that the Earth's figure has remained rigidly unchanged during such a period of time is certainly most improbable.

If the figure has been modified by the rotation, then the changes in the position of the axes resulting from the causes here considered would be on a still smaller scale. The secular changes resulting from the hypothesis would appear to be far too slow to produce any important effects even within geological periods.

I shall next suppose that the tidal action may be represented by a couple of resistance with its axis perpendicular to the plane of the ecliptic, and proportional to the relative angular velocity of the Earth and Moon on this plane. This supposition may be approximately true when the angular velocity is much reduced, and the truth must, I think, lie intermediate between the two hypotheses made.

In this case it will be sufficient to consider the Earth a sphere. If O z is the pole of the ecliptic, we have

$$A \frac{d\omega_3}{dt} = -f \{\omega_3 - n\}$$

$$\mathrm{A}\,\frac{d\,\omega_1}{d\,t} = 0. \qquad \mathrm{A}\,\frac{d\,\omega_2}{d\,t} = 0$$

$$\therefore \quad \omega_1 = \omega \cos \alpha \qquad \omega_2 = \omega \cos \beta$$

$$\frac{d\,\omega_3}{d\,t} + \mu\,\omega_3 = +\,\mu\, n$$

where

$$\frac{}{\mathrm{A}} = \mu$$

$$\therefore \quad \frac{d}{d\,t}\left(\epsilon^{\mu t}\,\omega_3\right) = \mu\, n\, \epsilon^{\mu t}$$

$$\omega_3 = \mathrm{P}\,\epsilon^{-\mu t} + n$$

$$\omega \cos \gamma = \mathrm{P} + n$$

$$\omega_3 = (\omega \cos \gamma - n)\,\epsilon^{-\mu t} + n$$

$$\Omega^2 = \omega^2 (\cos^2 \alpha + \cos^2 \beta) + \left\{n + (\omega \cos \gamma - n)\,\epsilon^{-\mu t}\right\}^2$$

And if λ is the angle between the pole of the axis of rotation and the pole of the ecliptic,

$$\cos \lambda = \frac{(\omega \cos \gamma - n)\,\epsilon^{-\mu t} + n}{\sqrt{\omega^2 \sin^2 \gamma + \left\{n + \omega\,(\cos \gamma - n)\,\epsilon^{-\mu t}\right\}^2}}$$

Therefore

$$\cos \lambda = \frac{\frac{n}{\omega} + \left(\cos \gamma - \frac{n}{\omega}\right)\epsilon^{-\mu t}}{\sqrt{\sin^2 \gamma + \left\{\frac{n}{\omega} + \left(\cos \gamma - \frac{n}{\omega}\right)\epsilon^{-\mu t}\right\}^2}}$$

Assuming as before that the whole 6″ in the secular acceleration of the Moon's mean motion in longitude is due to the change of rotation of the Earth, and taking $\gamma = 23^\circ$, $\frac{n}{\omega} = \frac{1}{28}$, and a million of years as the unit of time, we have

$$\text{Log}\ \mu = 5{\cdot}9301091.$$

The following numbers show the exceeding slowness with which the changes take place:—

For	$t = 0$	$\lambda = 23^\circ$
	$t = 1000$	$\lambda = 25^\circ$
	$t = 10000$	$\lambda = 43^\circ$
	$t = 100000$	$\lambda = 84^\circ\,45'$.

For $t = -100000$ the time of rotation on this hypothesis would be about 20 seconds.

The frictional action of the tides must, I conceive, lie between the two hypotheses made; the first being probably the more accurate for large velocities of rotation. On both suppositions we are led to secular displacements of the axis of rotation, but we cannot be justified in assuming the rigidity of the Earth during such periods as would be required to produce any considerable effects from the cause here considered. On the whole I am of opinion that this cause is not available for an explanation of those secular changes of climate which geologists have shown to have taken place on our Earth.

Observations of the Planet Mars. By John Joynson, Esq.

(*Communicated by John Stanistreet, Esq., F.R.S.*)

The observations of the planet *Mars* that have been made during the last three months have been almost entirely confined to the appearance of his disk. Every care has been taken not to examine the former observations while drawing the present appearances, to ensure as far as possible their being independently obtained, but they have resulted in confirming the former views to the fullest extent.

There can be no doubt whatever that the band is permanent, and that it extends all round the planet, with one, and as far as could be found, only one, narrow break in it. The colour of the band was generally a dark green.

The accompanying drawings are intended as a sequel to those sent in 1865 of the two previous oppositions.*

Waterloo, near Liverpool,
13 *February*, 1867.

On an Astronomical Presentiment of Immanuel Kant relative to the Constancy of the Earth's Sidereal Period of Rotation on its Axis. By A. D. Wackerbarth.

This great man's entire works (*Sämmtliche Werke*) were about a quarter of a century ago published at Leipzig, edited by Messrs. Karl Rosenkranz and Friedrich Wilhelm Schubert. The 6th volume contains his works on Physical Geography (*Schriften zur Physischen Geographie*), and opens with a little paper of 7 octavo pages, the title of which is: "Untersuchung der Frage; Ob die Erde in ihre Umdrehung um die

* These drawings were exhibited at the Meeting.